Complexities and simplicity: a review of agent-based artificial markets*

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Abstract. This paper reviews some agent-based artificial markets models classifying molecular, organizational and environmental sources of complexity. The first refers to the agents themselves, the second to the relationships among agents, the third to the market structures and rules in which traders are embedded. We describe a few examples for each category and argue that simplicity should have a big role at this point of the development of agent-based research. Two exemplar cases of terse models (molecular and environmental, respectively) are finally discussed and commented.
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1 Introduction

There is an interesting sentence impressed on a wall outside of the "Sloman" library of Essex University:

> The simple things you see are all complicated.

I believe this is a rather useful description of a (financial) market. Even without resorting to "invisible hands" or "hidden dictators", markets are simple and powerful devices capable to coordinate lots af economic agents, meet supply and demand, reveal information and so on. Yet, the market in itself is a complex object, where relevant institutional details interact with thousands of

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traders, possibly with diverse objectives, budgets, strategies, expectations, feedbacks and knowledge. The effects of intertwined structural properties and behaviour (broadly speaking) of agents can be stunningly complex at the macro level. The price time series, that might be regarded as the main result of trading, exhibit various degrees of leptokutosis, fat tails, heteroskedasticity, bubbles and crashes, to mention a few well-know and documented features, dubbed stylized facts. The same can be said for other byproducts of markets like volume, that is often considered excessive and intermittent or infraday bid-ask spread with its peculiar characteristics.

This review aims to point out some key issues in the agent-based artificial markets literature, with (a strongly biased) emphasis on the main ideas that are used and questions that are raised and answered. I'll try to keep under control the number of references in an effort to signal only the most important and clear-cut contributions.

Agent-based models can be very useful as many classic, often analytical, models fail to explain the empirical stylized facts observed in practice. Clearly no model is perfect and we should be prepared to observe some deviations from the predictions generated by simple modelling machinery. However, some key assumptions of standard models, like perfect rationality, the existence of a representative agent, unlimited foresight and computational power are questionable and might be relaxed in an attempt to describe single agents in a more credible way. This micro-simulation research avenue aims to explore the idea that a heterogeneous bunch of agents can collectively produce interesting and realistic phenomena. In other words, once we accept the idea that the market is not simple (or simplistic), we are immediately left with a huge number of ways to istantiate this non-simplicity[1].

One promising choice is to assume that there are a multiplicity of agents, endowed with individual traits affecting their behaviour. Clearly, this approach can quickly lead to the explosion of the parameters in the model and the researcher should endeavor to keep under control the complexity of the model (here proxied by the number of parameters) to enhance the understanding of causal relationships. Hence a fundamental question, especially in an agent based framework, is *"How much do I really need to explain this fact?"* which is possibly left unaccounted for in classic models. This appeal to simplicity is nothing more than a restatement of the Occam's razor principle: why should I use an intricate model if (almost) the same results can be obtained in a cleaner way? The temptation to complicate models is indeed strong in a agent-based, microscopically detailed framework but too rich models often make the results obscure as the effects can be attributed to a variety of causes. Hence, the previos question should be perhaps asked as "*Which is the minimal set of (individual) features needed to explain this (stylized) fact?"* I stress the need to strip down to the logical bones agent based models also for a more prosaic reason: after a decade of research in

[1] An example might clarify the argument: suppose linearity is rejected in a time series model. Then some non-linear feature has to be selected among a huge number of options (ARCH-GARCH, chaos, treshold models to mention a few).

the area, any serious attempt to improve the literature should not aim to provide "only" another source of fancy aggregate behaviour, as we often knows dozens of similar seeding mechanisms, but should adress the more difficult task to point to more fundamental issues.

After this argument in favour of simplicity, I would like to go back to the many possible sources of intricacies in a model. Inspired by a workshop on living technologies[2], I propose a classification of models in terms of different important features, dubbed "Molecular", "Organizational" and "Environmental" complexities.

The first refers to the complexity of the agent in itself: at one extreme, a trader can buy or sell randomly like in statistical mechanics-inspired models but agents can conversely have internal states, memory of their past actions, objectives, expectations, optmizing capabilities and so on.

The second complexity (*organizational*) describes the way agents perceive other agents and interact: some traders might be monads, insensitive to actions by others but, at the other extreme, agents might aggregate in groups or networks to share knowledge or strategies, learn from all or part of other market participants and receive global or local feedbacks.

Finally, agents are not living in a vacuum but they are embedded in a market environment, that is they have to trade using a specific set of rules dictating how and when they are allowed to exchange goods. The institutions are shaping and blending the behaviour of the agents that might be engaged in a old-fashioned bilateral trades or have access to cutting-edge electronic markets. It is crystalline that different market *environmental* architectures might have a big role in producing different aggregate results even if the agents (try to) behave in exactly the same way. In this respect, this complexity is linked to policy-making issues and may suggest how to to develop market institutions in order to obtain socially desired targets.

I believe that agent based models should be able to disentagle the effects of these complexities, stating what is the (main) responsible for the emergence of a stylized fact. This can be quite difficult in some occasions and often requires lenghty but almost unavoidable "robustness" analysis to check wheter the same effects can be obtained (almost) regardless of other details belonging to different complexities. To put things differently, assume that chartist, fundamental and noisy agents, that can change their mind and join in crowds from time to time while learning from each other, can produce several stylized facts in a batch auction market. So what? This model might be exciting and even realistic, but it's not really enhancing our understanding as there are too many diverse complexities involved. Are the results due to molecular, organizational or environmental features? Can we obtain the same results with simpler molecular agents, with simpler organizational structure? Are the results (only, mainly, not at all) due to the market environment?

[2] See the homepage of the European Centre for Living Technologies (ECLT) in Venice, `http://bruckner.biomip.rub.de/bmcmyp/Data/ECLT/Public`.

Some surveys provide fresh views of related literature and can greatly bolster the knowledge of artificial markets and agent-based models research fields. [22] is an early reading list with insightful comments and [23] is softly and neatly guiding newcomers to research. In [4] there is a nice review of the diverse areas that use agent-based methodology. Some of the cited papers are extremely stimulating though not connected to financial markets. An examination of agent-based models of organizations is presented in [19]. This survey is captivating as groups, firms and institutions intuitively display the multiscaled outcomes and richness of interaction between agents. A recent paper by Hommes ([20]) contains a detailed and comprehensive analysis of past research and is a very good introduction to the subject.

This paper is organized as follows: the following three sections are devoted to molecular, organizational and environmental complexity, respectively. In each section, some relevant papers are commented to clarify their main results and ideas. Observe that many agent-based models are discussing molecular, organizational and environmental features at the same time. Hence, despite the inclusion in one specific category might be driven by personal tastes, I discuss the paper in the section that is contributing the most to the message and results of the work.

These section should, as a whole, describe the richness of the agent based approach, that has demonstrated how complex macro-phenomena might be boiled down to simple micro features of the agents. The fifth section is about simplicity and reviews some simple models that are able to make some point in a clean way. Some conclusive remarks are given in the last section.

2 Molecular complexity

Generally speaking, agents are very often described as fundamentalist, chartist or noisy. The fundamentalists use some reference value to buy (sell) under- (over-)valued assets; chartists, on the contrary, estimate the prevailing trend in prices or use other technical rules to take profit of the sentiment of the market. Noise traders might act on random exogenous shocks or buy/sell assets randomly, regardless of fundamentals and trends. They are sometimes needed in order to provide liquidity to other agents using different strategies.

Zero Intelligence (ZI) agents are the focus of many essential papers, like the seminal work [17] or [13]. ZI agents are very simple agents, acting with minimal knowledge and sophistication. In a sense, they are neutral as their bareness should allow to shed light on other issues. [26] and [25] provide models with noisy traders or where fundamental and chartist agents can join the other group. All the previously mentioned papers will be discussed in the following sections, as their contribution is more clearly exposed in terms of organizational or environmental complexities.

An early example of papers where fundamentalists and chartists are present is [15]. The model influentially introduces a market where one informed, fundamentalist α-trader compete with a simplistic chartist β agent[3]. Agents never change their mind or trading strategies, submit to the market maker the quantities they want to buy/sell given the prevailing price and are never rationed. The market is modelled with a price-impact function that linearly adjust the current price depending on the excess demand. The paper mainly shows that price dynamics can be destabilized by these two opposite families of traders.

In [16], all agents trade when the common information they receive about returns exceeds a given personal threshold. The thresholds are updated randomly and asynchronuosly, the price formation mechanism is again based on a linear price-impact function and there is no communication or learning among agents. The resulting time series is quite realistic and is to be ascribed to the "threshold"-nature of the agents. An early interesting discussion of similar effects is in [18].

Quite complex agents are presented in [27]: traders have different amounts of money (according to a Zipf law), inclinations to investment, shares and lists of friends, expected gains, levels of tolerable loss and thresholds. Not surprisingly, the price time series generated by trading in a book-based market exhibits some stylized facts. However, it is not clear to me which is the more relevant effect discovered by the model. Complex agents from a molecular point of view are described also in [5].

3 Organizational complexity

Agents can organize in many different and important ways, through communication, imitation, learning, sharing of strategies and information and so on.

The paper by [25] shows that switching among fundamental-based and pessimistic/optimistic chartist rules can produce realistic time series. In detail, despite the fact that the relative changes of the fundamental values are normally distributed, the price dynamics is highly leptokurtic, fat tailed and persistent due to the profit-motivated shifts of agents among different strategies. The models in [8, 9] and [10], in a deterministic setting, clearly exhibit how fast switching among strategies can lead to chaotic price time-series. Hence, the hurried selection of strategies by agents can (and usually do) "destabilize" price.

This idea has some role also in the Santa Fe Artificial Stock Market (SFASM) described in [2], a much cited model where agents learn their strategies depending on continuos monitoring of profit, fundamental and technical parameters. Learning is based on a genetic algorithm that is run every k trading days. The key finding is that different regimes are triggered based on the rate of learning. If k is large, then the price is smoothly approaching a rational expectations equilibrium but if the genetic algorithm is invoked frequently then the price exhibits bubbles and crashes and a "psycologically rich" behaviour driven by the fluctuating fraction of traders that (temporarily) adopt technical trading. The effects of learning, modeled by a genetic algorithm, are relevant also in [1].

[3] This paper is still using representative α and β traders, but can be easily generalized to an authentic multi-agent framework with minimal effort (and changes on the results).

Another source of organizational complexity might lie in the ability of agents to have relationships with some partners in the market. Often, this is modelled assuming a network as the underlying structure of agents' acquaintances. In the paper by [14], agents sit at nodes in a random graph and each trader is randomly connected with probability p to other nodes. This mechanism is creating random connected components in the graph, that can be interpreted as coalitions of agents that submit the same kind of order (buy or sell) to the market. The price is adjusted adding a fraction of the excess demand and clearly the returns depend on the size of the connected components that are formed, which is in turn determined by the probability p. As some results in graph theory ensure that there are values of p such that the size of the components is fat tailed, this property is readily inherited by returns. The model has a simple and interesting special case: if $p = 0$, i.e. each agent is independently buying or selling, and no coalition is ever created, then the price is normally distributed due to the central limit theorem. As far as I know, this is the only case of a model where no stylized fact appears.

Similar ideas are developed in [21], where agents are arranged in a lattice and interact with neighbors. Hence the partners of one trader are individuated in a geographic-like space that might not have a physical counterpart but could be interpreted in terms of similarities in the information, trading techniques or objectives.

The investigation of network related agent based markets might profit of the underlying research about network models that has been booming in the last few years. This development could offer the modeler a host of results on the structure of agents relationships driving various macro effects on price.

4 Environmental complexity

The impact and importance of the market architecture has been acknowleged only recently in the agent-based framework. Neglecting the precise details of the price formation mechanism was (and still is) indeed common in many papers about artificial markets. A moment of reflection, however, shows that many peculiarities of prices might be an almost direct consequence of the way trading is performed. This might be due to the hardness of an accurate modelization of actual markets. The relevance of market structures is of great importance for policy issues. Given that it is virtually impossible to enforce "proper" behaviour in selfish agents, it is still possible to get some socially desired results by acting on the rules that regulates the market. We might argue that devising a clever trading architecture is really an obligated avenue for governments or regulatory agencies that might like, say, low volatility, high allocative and informational efficiency, price stability and so on. It might be true that some architectures (batch auctions or order-book markets or dealerships to exemplify) could achieve one of the aforementioned targets better than other ones.

The importance of the market was strongly emphasized in [17] where Zero Intelligence (ZI) traders place random limit orders with the basic restriction that they cannot buy above the value or sell below the cost of the item. This unique limitation, coupled with a batch market, is enough to obtain almost perfect allocative efficiency. Hence, given that the agents are behaving as dolts, the result is attributable to the market alone[4].

A comparison of different markets is presented in [6] and [3]. The first paper is a comparison of three trading protocols, namely a walrasian tatonnement, a batch auction and an order-book market. The agents are myopic quadratic utility maximizers that adjust their portfolios using chartist or purely noisy forecasts of mean and variance of returns. A grid of ecologies of traders, indexed by the fraction of chartist-type agents, is studied. The comparison shows that the statistical propertyies of returns are “largely shaped by the specific architecture of the trading mechanism” but depends also on the trader ecology. In other words, results are due simultaneously to environmental and molecular features.

The paper by Audet and coauthors studies retail agents that can trade on two market platforms, to wit a two-level dealership and batch auction. Agents are exponential utilility maximizers, are forced to rebalance their holdings by exogenous shocks of varying intensity and learn their strategy using a neural net given that all competitors use their best response. The preferred market should allow the agents to maximize their final wealth. The results again depends both on the agents and the dealers: roughly speaking, the dealership is preferred in a thin market (because traders can find more convenient quotes) or when agents are affected by large liquidity shocks.

The aftermaths of a book-order market is indirectly claimed in [26] and [24]. In the first paper, agents submit purely random limit orders while in the latter fundamentalist traders try to buy (sell) under- (over-)valued assets on a subjective risk-adjusted basis. The argument basically goes along the following lines: as the agents are behaving so simply (no learning, no herding. . .) the resulting properties of returns can be imputed to the market protocol. Succinctly, order-book markets robustly generates fat tailed and leptokurtic returns (this conclusion is especially compelling in Maslov’s work that will be analyzed in the following section because of its notable terseness).

We conclude this section mentioning some work contained in [11, 12] that develops the exciting environmental idea that markets can coevolve with agents. The consideration that changes of the market protocols should trigger adjustments of selfishly profit-seeking agents is of capital importance as disregarding the reaction by agents might make any reform worthless or even harmful.

5 Simplicity

This section examines two models that show remarkable simplicity and make a clear point with exemplar terseness. We believe these models are especially tailored to be used as pedagogical benchmarks and provide good starting points

[4] If you do not like the idea that the market is the only important thing to get efficiency, some interesting development can be found in [7], that shows cases where humans can reach efficiency while ZI agents cannot.

to develop a sound comprehension of agent-based artificial markets. As mentioned previously, the need for simplicity is very strong in this research field, as the possibilities to muddle models are really innumerable with a serious risk to obscure the main message in a forest of possibly irrelevant details.

The paper by [16] presents a market where all agents, indexed by $i = 1, \ldots, N$ receive the same random information ϵ_t at time t. They must decide to buy, sell or stay put setting $\phi_i(t) = +1, -1, 0$, respectively. The trading strategy of each agent is based on a subjective time varying threshold $\theta_i(t)$ that is used to interpret the informative signal ϵ_t, that should be strong enough to trigger further action. One agent buys if $\epsilon_t > \theta_i(t)$, sells if $\epsilon < -\theta_i(t)$ and does nothing otherwise. Two more ingredients are needed to close the model, namely threshold adjusting and price formation mechanisms. Agents independently change their θ's to the last return $r_t = \log(p_t/p_{t-1})$ with some probability s per trading day. Hence, they raise their activation thresholds in more volatile markets and lower them in quiet periods. The price change is readily obtained using a common (though debatable) device: if $Z_t = \sum_i \phi_i(t)$ is the excess demand at time t then

$$r_t = \log \frac{p_t}{p_{t-1}} = g\left(\frac{Z_t}{N}\right),$$

where $g(z)$ is an impact function that is assumed linear in the simulations, $g(z) = z/\lambda$. The model can be summed up as follows, dropping t for simplicity.

1. Agents receive a common normally distributed signal ϵ;
2. Each agent compares the signal with his/her threshold θ_i, setting correspondingly the demand ϕ_i to $+1, -1$ or 0;
3. The excess demand Z is computed and used to obtain the current price p_t;
4. Finally, all agents change their θ's with probability s, i.e. if it is the case they set $\theta = r_t$.

The core of the model can (exceptionally!) be written in a few lines of code with `R` and the fragment of program is listed below[5].

```
   for(time in 1:epochs){
1  phi <- 0
2  epsilon <- rnorm(1,0,d)
3  phi[epsilon>theta] <- 1
4  phi[-epsilon>-theta] <- -1
5  z <- sum(phi)
6  price <- price*exp(z/N/lambda)
7  changing <- runif(N)<s
8  theta <- ifelse(changing,z/N/lambda,theta)
   }
```

In line 2, a random normal signal is drawn. Lines 3-4 set the demand to +1 or -1 if appropriate. The price p_t is computed in line 6, based on the excess

[5] See `http://cran.r-project.org` for information about `R`.

demand Z. Lines 7-8 implement the asynchronous updating of the thresholds. `changing` is a boolean vector flagging the agents that should update θ that is set to $r_t = Z/(N\lambda)$ with probability s or left unchanged. A complete simulation of Cont's model just requires some additional trivial inizialization and data-collecting instructions.

Despite its striking[6] simplicity, the model shows a host of stylized facts. All the results can nicely be ascribed to a unique molecular source, namely the changing threshold-based activity of agents.

The model in [26] is our second example of simplicity. Non-strategical agents can randomly place market or limit unit orders in a book-based framework. In detail, at time t an agent who is equally likely to be a buyer or a seller can submit a limit (market) order for one unit with probability q_{lo} $(1 - q_{lo})$. If a limit order is issued, the corresponding limit price is obtained offsetting the prevailing price $p(t)$ by an amount Δ, distributed as $P(\Delta)$. Specifically, on the buy (sell) side an order with limit price $p(t) - \Delta$ $(p(t) + \Delta)$ is submitted. As the author notes, the behaviour of the agents is completeley passive and mechanical (in addition to its purely random character).

This model is even simpler than the previous one if agents are considered (I wonder how they could be simpler), yet the focus is on the book-based market that is a more realistic protocol and cannot be coded trivially in a few lines. Simulated price time-series are not realistic but exhibit fat tails, heteroskedastic effects and long term correlations. Despite its minimalistic molecular features, the environmental trait represented by a limit order-driven market is reproducing alone some stylized facts. As far as we know, this is one of the first works in artificial markets literature, another being [17], where the importance of the trading architecture is given prominence as the main ingredient of the results.

6 Conclusion

Agent-based microsimulation of financial markets has seen almost a decade of impetuous success. Many empirical regularities and stylized facts that are hard to explain in a classic representative framework can be quite successfully modelled and tracked down to micro-features of the agents. This survey argues that the results can be mainly due to three fundamental sources, namely molecular, organizational and environmental complexieis. These different facets can be appealing in different research areas as, say, policy-makers could be more interested in environmental issues, behaviorist scholars in molecular specificities and sociologists might be particularly fond of organizational details. The degrees of freedom in agent-based models are however so many that there is the risk to obscure the significance of the model, especially if multiple complexities are acting at once. Some simple and terse models are discussed showing with no (or little)

[6] The model is not without pitfalls: a moment of reflection shows, for example, that at any time t, all active agents are taking the same position, which means that all active traders are buying or selling. This is clearly pathological but we are not searching for realism that could be easily added in the model incorporating further components.

ambiguity that specific targets can be addressed in a clean way. For example, many stylized facts can be linked to molecular threshold trading rules used by agents, as in [16]. Similar effects could also be generated by an environmental order-driven market, as shown in [26].

The fact that the same phenomena can be described and understood in fundamentally different ways should foster further research to clarify the point in the search of ideal and pithy models.

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